4.1. A salesman flies around between Atlanta, Boston, and Chicago as follows.

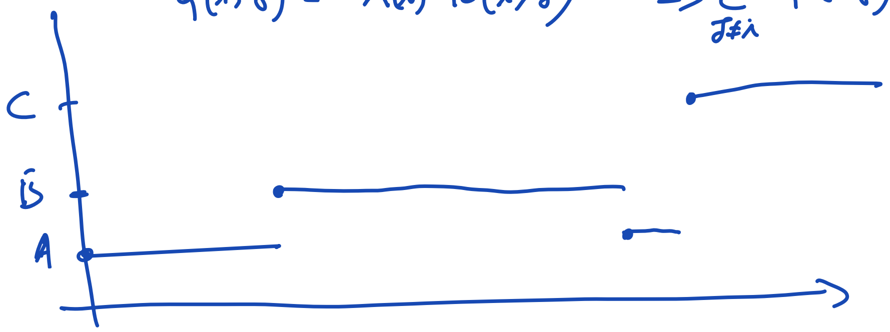
$q(A,B)+q(A,C) = \lambda(A)$

$$\begin{array}{c|ccc} & A & B & C \\ \hline A & -4 & 2 & 2 \\ B & 3 & -4 & 1 \\ C & 5 & 0 & -5 \end{array}$$

$$Q = \begin{pmatrix} -\lambda(A) & q(A,B) & q(A,C) \\ q(B,A) & & q(B,C) \\ q(C,A) & q(C,B) & \end{pmatrix}$$

(a) Find the limiting fraction of time she spends in each city. (b) What is her average number of trips each year from Boston to Atlanta?

$q(i,j) = \lambda(i)\cdot r(i,j) \Rightarrow \sum_{j\neq i} q(i,j) = \lambda(i) \underbrace{\sum_j r(i,j)}_{=1} = \lambda(i)$



What are the holding rates $\lambda_A, \lambda_B, \lambda_C$

$\lambda(A) = 4 \qquad \lambda(B) = 4 \qquad \lambda(C) = 5$

$$r(A,B) = \frac{q(A,B)}{\lambda(A)} = \frac{2}{4} = 1/2$$

$$q(A,B) = \lambda(A)\, r(A,B)$$

$$R = \begin{pmatrix} 0 & \frac{2}{4} & \frac{2}{4} \\ 3/4 & 0 & 1/4 \\ 1 & 0 & 0 \end{pmatrix} \qquad Q \; \begin{array}{c|ccc} & A & B & C \\ \hline A & -4 & 2 & 2 \\ B & 3 & -4 & 1 \\ C & 5 & 0 & -5 \end{array}$$

the proportion of time spent in Atlanta

$\pi(A)$

$\Rightarrow$ I have to compute π by solving

$$(\pi_1, \pi_2, \pi_3) \begin{pmatrix} -4 & 2 & 2 \\ 3 & -4 & 1 \\ 5 & 0 & -5 \end{pmatrix} = (0\ 0\ 0)$$

$$P(X_t = B \mid X_0 = A) = p_t(A, B)$$

$\Rightarrow$ I have to solve the following eq.

$$p_t'(i,j) = \sum_{k \neq j} p_t(i,k) q(k,j) - p_t(i,j)\lambda(j)$$

$$p_t'(A,B) = p_t(A,A) q(A,B) + p_t(A,C) q(C,B) - p_t(A,B) \cdot \lambda(B)$$

$$= p_t(A,A) \cdot 2 + p_t(A,C) \cdot 0 - p_t(A,B) \cdot 4$$

$p_t'(A, c) = \cdot \quad \cdot \quad - \quad -$

$\cdot \quad - \quad -$

$$P_t = \exp(Qt) = \sum_k \frac{(Qt)^k}{k!}$$

$$P \begin{pmatrix} 1 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 1 \end{pmatrix} \quad Q \begin{pmatrix} & 0 & 0 \\ \cdot & \cdot & \cdot \\ 0 & 0 & \end{pmatrix}$$